

# Mathematical model of layered metallurgical furnaces and units

V S Shvydkiy, N A Spirin and V V Lavrov

Department of Thermophysics and IT in Metallurgy, Ural Federal University, 19 Mira Street, Yekaterinburg, 620002, Russia

E-mail: vshvit@isnet.ru

**Abstract.** The basic approaches to mathematical modeling of the layered steel furnaces and units are considered. It is noted that the particular importance have the knowledge about the mechanisms and physical nature of processes of the charge column movement and the gas flow in the moving layer, as well as regularities of development of heat- and mass-transfer in them. The statement and mathematical description of the problem solution targeting the potential gas flow in the layered unit of an arbitrary profile are presented. On the basis of the proposed mathematical model the software implementation of information-modeling system of BF gas dynamics is carried out. The results of the computer modeling of BF non-isothermal gas dynamics with regard to the cohesion zone, gas dynamics of the combustion zone and calculation of hot-blast stoves are provided

## 1. Introduction

One of the main problems in the field of development of the theory of layered metallurgical processes is the maximal use of the units capacities, thermal and reducing potentials of gases and, ultimately, achievement of highest furnaces productivity and minimal fuel consumption.

From the point of view of this problem of utmost importance has the mathematical description of the physical, mechanical, thermal and chemical processes that occur in the layered metallurgical furnaces and units, creation of mathematical models of machines of various application.

It is to be recalled that the mathematical model of the object is usually understood as the simplified representation of processes in the unit, which preserves the most essential properties of the real object and transfers them in the mathematical form. In this sense we can speak about the homomorphism of the object (of the unit or its components) in its model. Depending on the task the mathematical model takes into account a different number of properties and attributes of the object, and therefore can be wide or narrow. The wide model more fully and more accurately than the narrow one reflects the properties, but, as mentioned above, it is only an idealized reflection of the real object. In a certain area of the model these model are homomorphic, i.e. may provide similar or identical results.

Mathematical model of the layered metallurgical furnace or unit must be, on the one hand, easy enough to visually and distinctly transmit all the qualitative aspects of the phenomenon we are interested in (only in this case it is possible to maintain “physical control” over the used model), and, on the other, precisely enough to pass the quantitative patterns of process flow. These requirements are in conflict, because without a thorough study of the system properties it is not clear which factors are



important. The only rational compromise can be found only within a hierarchical approach, suggested by the very nature of a layered unit.

Layered metallurgical furnaces and units, being complex objects, have a multi-stage structure, and their mathematical models should be built consistently on the basis of preliminary building of components models. The research of a complex process by parts makes it possible to move to a higher level model, including into it as constituent a narrow pattern of a lower level. However, this transition will be correct unless a number of conditions are fulfilled, namely:

- the whole range of change in the dependent and independent variables, we are interested in, are included into a broader model of lower-level models, is known (including the boundary conditions);
- the narrow model in this entire range is analyzed;
- it is shown that the narrow model adequately describes the process at a lower level.

The indicated method of constructing mathematical models of a layered unit on levels suggests that during the model building of this level all essential physical and chemical laws, which determine the properties of this level, are deeply studied and experimentally verified. In this case, the patterns acquire a predictive power of physical laws; they are invariant in space and autonomous over time. Hence follows the principle of invariance of the model components to the geometric dimensions of the system and the autonomy of these parts over time [1]. It can be formulated as follows: Regularities of processes in the component parts of the models of this level, as well as regularities of interaction between these parts can be expressed in a form that does not depend on the scale of the level in question and the point of time. It should be noted that in this definition we are talking about invariance and autonomy of the processes regularities at the lowest level to the processes at the highest level, but not about the invariance and autonomy of quantitative dependences.

For a correct transition from the lowest level to the highest one invariant relative to the scale and autonomous over time should remain not only the regularities, but also the parameters in the mathematical description of the components of the highest level and in the boundary conditions. This independence of regularities and parameters on the scale of level and time indicates their objectivity and allows the process in units of almost any type and size to be predicted since these parameters determine the value of similarity numbers of processes in units of various sizes.

The successive building of a mathematical model from the lowest to the highest level on the basis of invariance and autonomy principles of the model constituent parts opens the possibility of independent experimental determination of the unknown parameters of this model in the conditions, when, firstly, there is a greater sensitivity of the process to these parameters (and therefore ensures a high precision of the experiments), and, secondly, the influence on the process by the most part of other factors is practically excluded.

The building of the mathematical models on the basis of these principles does not exclude, but presupposes the mathematical procedure of enlargement, “compression” of the information about any part of the level, presenting it in a compact, convenient for the subsequent analysis form. In this from the mathematical model may lose its physical visualization, may appear and appear the “signs of empiricism”, and the coefficients become “effective”.

Thus, the division of a complex process in layered metallurgical furnace or unit into its constituent parts must be preceded by a thorough study of the nature of physical and chemical phenomena, which, as a result, further leads to the work not only with mathematical expressions, but with independent from the scale and time physical concepts. Reliability and accuracy of the mathematical models and the results obtained on their basis depend on the level of development of these ideas. The knowledge about the mechanisms and physical nature of the charge column movement and the gas flow in the moving layer, as well as the patterns of development of heat- and mass-transfer in them, has the particular importance when it is applied to the layered metallurgical furnaces and units.

A detailed overview of the current state of the theory of thermal processes in the metallurgical units with a dense layer with an extensive bibliography is presented in monographs [2-4]. Therefore, here

we formulate only the main conclusions resulting from the indicated review and largely determining the content of this paper.

1. Despite the abundant experimental data and physical substantiation of the mechanism of granular media flow through the hole, up to now there is no sufficient logical and reliable mathematical model describing the material movement in the shaft furnaces.

2. The experimental studies of the layer gas mechanics continue to develop rapidly. New methods of measurement of velocities and gas pressure in the layer are offered, which allowed the processes of gas motion in real furnaces to be studied, and numerous experiments, carried out on the ovens and models, made it possible to clarify the dependence of pressure drop on gas velocity and to identify the influence of various parameters of shaft furnaces on gas distribution in them, and also offer a variety of methods for gas flow control. However, the experimental methods are characterized by the well-known disadvantages (great labour intensity, complexity of the conditions existing in the shaft furnaces and complicating the research, difficulties in building of a large variety of physical models of furnaces, etc.) that make use the calculation and theoretical investigations of the layer gas mechanics.

3. Currently, the calculation and theoretical investigations of gas movement in the in general and in the blast furnaces in particular are developing very intensively; in the world several metallurgical centers are engaged in the mathematical description of gas mechanics of the lump materials layer. The model of the gases movement in the layer, offered by the research team of the Department of Thermophysics and IT in Metallurgy, Ural Federal University, Russia (see the indicated above references) are characterized by its completeness.

4. The theory of heat-transfer in the layer achieved a great success. Today it can be stated [2–4, 5], that the one-dimensional problem of heat-transfer in the layer is almost exhausted. At the same time it should be taken into account that in real layered metallurgical furnaces and units the gases movement is usually multidimensional. The uneven distribution of gas velocity in the layer volume and along the cross-section of units is quite substantial.

The problem of the development of complex mathematical models of layered metallurgical furnaces and units consists in derivation of the gas flow equation for moving layer, connecting the average velocity in the inter-lump area (true speed), its actual pressure and temperature, as these values influence the intensity of flow of physical and chemical transformations. At the same time the laws of conservation of mass, quantity of motion and energy should be fulfilled. Apparently, the solution to this problem should be based on the achievements in the modern theory of continuous media [6] using the parallels between the gas flow in the layer and the movement of a hypothetical fluid, which occupies the entire volume of the unit, including the volume of lump materials.

The consistent satisfaction of the requirements of dynamic properties equality of this liquid to the local averaged parameters of real heterogeneous medium (aggregate of lump materials and gases) makes it possible to reduce a complicated problem of gases movement in the layer of variable structure to a relatively simple problem of the homogeneous liquid movement, overcoming, however, some additional resistance.

## **2. Averaged continuity equations of motion and gas energy in the layer**

A detailed derivation of all equations of layer gas mechanics is given in [2–4], so here we shall only illustrate the realization of the gas flow analogy in the real layer and a hypothetical homogeneous liquid on the example of continuity equation. The remaining equations we shall provide without conclusion.

So, let the shaft furnace of an arbitrary profile be filled with lump material of porosity  $\varepsilon$  and cross-sectional pore aperture  $\varepsilon_p$ . In general  $\varepsilon$  and  $\varepsilon_p$  are the functions of the form of the charged lumps and coordinates. In the bottom the gas is injected through the tuyere or otherwise. The medium thermal properties are the functions of pressure, temperature, and coordinates. The gas gives the material its heat and can chemically interact with it. It is required to find the distribution of velocities, pressures, temperatures and concentrations of gas components along the height and cross-section of the furnace.

Let us distinguish in the layer an elementary volume  $V$  ( $\text{m}^3$ ). In this volume the gas occupies  $V'$  ( $\text{m}^3$ ). If its density  $\rho$  ( $\text{kg}/\text{m}^3$ ), the mass of the gas in the indicated volume will be  $\rho V'$  (kg).

Let us consider the mass balance in the element of volume  $V$ . Change in the gas mass in this amount equals to

$$\frac{d}{d\tau} \int_{V'} \rho dV' = \int_V \frac{\partial}{\partial \tau} \left( \rho \frac{dV'}{dV} \right) dV = \int_V \frac{\partial}{\partial \tau} (\rho \varepsilon) dV, \quad (1)$$

where  $\varepsilon$ — average porosity of layer in the volume  $V$ :

$$\varepsilon = \frac{dV'}{dV} = \frac{1}{dV} \int_V \varepsilon_n dV, \quad (2)$$

where  $\varepsilon_n$ — local value of porosity.

Mass change equals to its flow through the closed surface  $S'$  (contour) of the volume  $V'$ , and the full flow of mass in this case is

$$\oint_{S'} (\rho \vec{v}) d\vec{S}' = \oint_S (\rho \vec{v}) \left( \frac{dS'}{dS} \right) d\vec{S} = \oint_S (\rho \varepsilon_n \vec{v}) d\vec{S} = \oint_S (\rho \varepsilon_n \vec{v}) \cdot \vec{n} dS. \quad (3)$$

Here  $\varepsilon_p$ — average cross-sectional pore aperture of the layer in the volume  $V$ , and  $n$ — normal to a surface of contour  $S'$ .

Taking into account the effect of inner sources (sinks) of mass with capacity  $q_V$  (the mass formation due to the physical and chemical transformations), after conversion of a surface integral into the volumetric one according to the divergence theorem [7-9], we obtain

$$\int_V \left[ \frac{\partial}{\partial \tau} (\rho \varepsilon) + \nabla \cdot (\rho \varepsilon_n \vec{v}) - q_V \right] dV = 0, \quad (4)$$

from it because of the arbitrariness of volume  $V$  we find

$$\frac{\partial}{\partial \tau} (\rho \varepsilon) + \nabla \cdot (\rho \varepsilon_n \vec{v}) = q_V. \quad (5)$$

Comparing equation (5) with continuity of hypothetical continuous flow [10-12]

$$\frac{\partial \rho_*}{\partial \tau} + \nabla \cdot (\rho_* \vec{v}_*) = q_V, \quad (6)$$

we find, that relations (5) and (6) become equivalent, if the equalities are satisfied

$$\rho_* = \varepsilon \rho; \quad \vec{v}_* = (\varepsilon_p / \varepsilon) \vec{v}. \quad (7)$$

Thus, as it is shown in [13, 14], the physical meaning of equations content (7) is as follows. To describe the processes occurring in the layer, it possible to use the equations derived for a continuous medium, if we know the effective values of its thermal parameters. It is obvious that these values are

not equal to the values of real gas parameters. We should emphasize the fact that any mathematical model of gas motion in the layer describes the patterns of change in average characteristics of the flow field.

During derivation of motion equations Cauchy stress principle is usually taken as a postulate of [5, 6, 10], stating that for any closed surface  $\Omega$  there is a distribution of voltages vector  $\mathbf{t}$  with the resultant and the moment equivalent to the forces field influencing on the continuous medium, inside  $\Omega$ , from the medium side located outside this surface. It is assumed that at a given moment of time vector  $\mathbf{t}$  depends only on the position and orientation of the surface element  $dS$ , i.e. if we denote by  $\mathbf{n}$  the outer normal to the surface  $\Omega$ , then  $\mathbf{t} = \mathbf{t}(\mathbf{x}, \tau, \mathbf{n})$ .

Let us now recall the basic dynamics principle of fluid motion, called the principle of momentum conservation [5, 6, 10]: rate of change in fluid movement quantity, enclosed in the moving volume  $\Sigma$  equals to the resultant of the forces acting on the fluid.

The analytical expression of this principle is the equation

$$\frac{d}{d\tau} \int_{\Sigma} \rho_* \bar{\mathbf{v}}_* dV = \int_{\Sigma} \rho_* \bar{\mathbf{f}} dV + \int_{\Omega} \bar{\mathbf{t}}_* dS, \quad (8)$$

where  $\mathbf{f}$ – field of external forces attributed to the unit mass.

Averaging this equation for the volume of  $V$ , as we did it for the mass balance equation (for details see [4]), we arrive at an intermediate form of the motion equations of

$$\begin{aligned} \varepsilon \rho \frac{d}{d\tau} (\varepsilon_n \bar{\mathbf{v}} / \varepsilon) = \varepsilon \rho \bar{\mathbf{g}} - \text{grad}(\varepsilon_n p) - \frac{2}{3} \text{grad}[\varepsilon_n \mu \text{div}(\varepsilon_n \bar{\mathbf{v}} / \varepsilon)] + \\ + 2 \cdot \text{div}(\varepsilon_p \mu \mathbf{D}) + \mathbf{R}. \end{aligned} \quad (9)$$

Here  $p$ – gas pressure,  $\mu$  – coefficient of dynamic viscosity of gas,  $\mathbf{R}$  – additional resistance that occurs during the transition from real gas to homogeneous fluid, and  $\mathbf{D}$  – strain velocity tensor with components

$$D_{ij} = 0.5(\partial v_i / \partial x_j + \partial v_j / \partial x_i), \quad (10)$$

where  $i, j = 1, 2, 3$ ;  $x_1 = x, x_2 = y, x_3 = z$ ;  $v_1 = u, v_2 = v, v_3 = w$ . Note, that in equation (9)  $d/d\tau$  characterizes the so-called *full* or *substantial* derivative, i.e.  $d/d\tau = \partial/\partial\tau + u(\partial/\partial x) + v(\partial/\partial y) + w(\partial/\partial z)$ .

The last summand of the right-hand side (9), which represents additional resistance to the gas movement through the layer, at the present level of knowledge can not be expressed in quadratures. The only way to determine the value of  $\mathbf{R}$  is an experiment. It is clear, however, that the resistance  $\mathbf{R}$  should be a unique function of layer porosity, i.e.  $\mathbf{R} = \mathbf{R}(1 - \varepsilon)$ , since in the absence of solid particles ( $\varepsilon = 1$ ) the value  $\mathbf{R} \equiv 0$ . In [2-4, 15-17] various approximations of the results of experimental determination of  $\mathbf{R}$  are discussed. In [4] it is stated that the most reliable dependence for operation of shaft metallurgical furnaces is the following:

$$\begin{aligned} \mathbf{R} = -2 \text{div}(\varepsilon_p \mu \mathbf{D}) + (2/3) \text{grad}[\varepsilon_p \mu \text{div}(\varepsilon_p \mathbf{v} / \varepsilon)] - 150 \left( \frac{1 - \varepsilon}{\varepsilon d_k \Phi} \right)^2 \mu \frac{\varepsilon_p^2}{\varepsilon} \mathbf{v} - \\ - 1.75 \left( \frac{1 - \varepsilon}{\varepsilon d_k \Phi} \right) \rho \frac{\varepsilon_p^2}{\varepsilon} \left( \frac{\varepsilon_p}{\varepsilon} |\mathbf{v}| \right) \mathbf{v} \end{aligned} \quad (11)$$

Here  $\Phi$  is the particle shape factor of the material (1 for a sphere);  $d_k$  – particle diameter.  
 The last two summands of relation (11) for  $\mathbf{R}$  define the so-called law of resistance of S. Ergun.  
 Using (11) the equations of motion (9) are simplified and have the form

$$\varepsilon \rho \frac{d}{d\tau} \left( \frac{\varepsilon_p}{\varepsilon} \mathbf{v} \right) = \varepsilon \rho \mathbf{g} - \text{grad}(\varepsilon_p p) - \left[ 150 \left( \frac{1-\varepsilon}{\varepsilon d_k \Phi} \right) \mu + \frac{\rho \varepsilon_p}{1.75 \varepsilon} |\mathbf{v}| \right] \frac{1-\varepsilon}{\varepsilon d_k \Phi} \frac{\varepsilon_p^2}{\varepsilon} \mathbf{v}. \quad (12)$$

Instead of S. Ergun equation any experimental dependence can be used [2-4, 15-17].

The energy equation is derived by the similar method described in [4]. In this case, two circumstances must also be taken into account. Firstly, the heat-transfer by thermal conductivity is carried out not only in the gas, but from the gas to the surface of material particles. As the evaluation of the last component of heat-transfer requires data on identification of patterns of development of thermal and dynamic boundary layers, it is advisable to replace this summand by Newton's law of heat-transfer by convection. Secondly, the presence of additional force of resistance in the equations of motion leads to the appearance in the energy equation of a summand, characterizing the transition of mechanical energy into thermal energy. Omitting the intermediate calculations (for details see [2-4]), we shall write the final result

$$\varepsilon \rho C_v \frac{dT}{d\tau} = \varepsilon \rho q_R + \text{div}(\varepsilon_p \lambda \text{grad} T) - \alpha_v (T - t) - \varepsilon_p p \text{div}(\varepsilon_p \mathbf{v} / \varepsilon) - \\ - (2/3) \varepsilon_p \mu [\text{div}(\varepsilon_p \mathbf{v} / \varepsilon)]^2 + 2 \varepsilon_p \mu \sum_{i,j} D_{i,j}^2 + q_g', \quad (13)$$

where  $C_v$  – is gas heat capacity (kJ/(kg·K)),  $\lambda$  – coefficient of gas thermal conductivity (W/(m·K)),  $\alpha_v$  – volumetric heat-transfer coefficient (W/(m<sup>3</sup>·K)),  $q_R$  – velocity of heat inflow due to radiation (W/kg),  $q_g'$  – source (sink) of heat acting in the gas (W/m<sup>3</sup>). This summand, as noted before, also includes the dissipation of mechanical energy occurring on the surface of material particles.

The first three summands of the equation right-hand member (13) characterizes three types of heat-transfer: radiation, conductivity and convection. For the operation conditions of metallurgical shaft furnaces the thermal conductivity of gases can be ignored [2]. Typically the gas thermal conductivity is taken equal to zero in the direction of its motion and infinity – across the flow. Since the effective beam length in the inter-lump space is insignificant, and the weakening capacity of dust-laden gas flows is great, the radiative transfer cannot also be taken into account. This assumption is especially justified by the fact that the heat-transfer coefficient  $\alpha_v$  is usually determined experimentally. For this reason, the radiant component, which is difficult to calculate, is included into  $\alpha_v$  through the dependence of the latter on temperature. However, it should be noted that this simplification does not lead to noticeable errors only for areas of the layer in which the coolant temperature does not exceed 500 °C [9].

The following three summands of the equation right-hand member (13) described the work of the pressure forces and the dissipation of mechanical energy (gas dynamic heat). These values become comparable with the convective heat-transfer only at the sub- and supersonic gas velocities. The shaft metallurgical furnaces do not have such velocities, then without any losses for the of calculations accuracy these summands can be neglected.

Thus, for the conditions of heat and gas dynamic operation of shaft metallurgical furnaces the energy equation transforms into the equation of heat-transfer

$$\varepsilon \rho C_v \frac{dT}{d\tau} = (\varepsilon \rho q_R) - \alpha_v (T - t) + q_g, \quad (14)$$

where  $q_g$  sources (sink) of heat conditioned by the physical and chemical reactions proceeding in the gas. The first summand in the right-hand member is placed in brackets in order to emphasize the need for use of numerical methods of calculation when taking into account the radiation heat-transfer.

For system closure it is necessary to add the equation:

– gas condition

$$\rho = \rho(T, p); \quad (15)$$

– heating of material lumps

$$(1 - \varepsilon)\rho_m C_m \frac{dT}{d\tau} = \alpha_v(T - t) + q_m; \quad (16)$$

– regularities of change in gas viscosity and heat-conductivity

$$\mu = \mu(T, p); \quad \lambda = \lambda(T, p), \quad (17)$$

–and mass-transfer

$$\frac{dC}{d\tau} = -K_\Sigma f[C(1 + \bar{K}) - \bar{K} \cdot \Sigma] \quad (18)$$

and

$$\frac{d\varphi}{d\tau} = n \frac{dC}{d\tau}. \quad (19)$$

In these expressions  $\rho_m$  – material density ( $\text{kg/m}^3$ ),  $C_m$  – specific heat capacity of the material ( $\text{kJ}/(\text{kg}\cdot\text{K})$ ),  $q_m$  – sources (sinks) of heat acting in the material ( $\text{W/m}^3$ );  $C$  – concentration of a reductant in the gas ( $\text{m}^3/\text{m}^3$ ),  $\bar{K}$  – equilibrium constant,  $K_\Sigma$  – effective constant of reaction rate ( $\text{m/s}$ ),  $f$  – value of the reaction surface ( $\text{m}^2/\text{m}^3$ ),  $\Sigma$  – equilibrium concentration,  $\varphi$  – degree of oxide reduction (fractions of a unit),  $n$  – ratio of gas flow mass-capacity  $W_g^*$  to material flow  $W_m^*$ .

Due to the substantial nonlinearity of equations (5), (12), (14) – (19) their joint solution can be done only numerically. For this reason, to obtain a sufficiently reliable result it is advisable to use the iterative method. First, for the arbitrarily given initial temperature field, the problem of layer gas mechanics is solved, then the heat- and mass-transfer equations. The iterations are carried out up the specified accuracy of calculations of temperature fields in the layer. From a practical point of view it is better not to use the equation of motion (12), but more convenient simplified expressions that take into account peculiarities of gas movement in the layer for a specific unit.

### 3. Possible simplifications of a gas mechanics model of the moving layer

*Potential flow.* One of the possible simplifications of the model is the assumption about the potential (non-vortex) nature of the gas flow. The mathematical condition of the vorticity absence is the ratio of  $\text{rot } \mathbf{v} = 0$ . From the field theory [7, 8] it is known that if this condition is satisfied, then there is a scalar function  $\varphi(\mathbf{x})$ , called the *potential function*, gradient of which equals to the velocity  $\mathbf{v}$ , i.e.

$$\mathbf{v} = \text{grad}\varphi. \quad (20)$$

The minus mark means that the flow moves from a larger to a smaller potential. Inserting the expression (30) into the equation (5) we obtain the equation for finding the potential function  $\varphi$ :

$$\operatorname{div}(\varepsilon_p \rho \operatorname{grad} \varphi) = 0. \quad (21)$$

The components of gas velocity are calculated according the known field  $\varphi$  using relation (20). Knowing the velocities values on the basis of equations the motion of gas pressure can be determined. For non-vortex flows equation (12) takes the form:

$$-\operatorname{grad}(\varepsilon_p p) = \rho \varepsilon_p \operatorname{grad}(q^2/2) - \rho \varepsilon_p [\mathbf{v} \operatorname{grad}(\varepsilon_p/\varepsilon)] + (A_1 + A_2 q) \mathbf{v} - \rho \varepsilon \mathbf{g}. \quad (22)$$

Here

$$A_1 = 150.0 \left[ \frac{(1-\varepsilon) \varepsilon_p}{\varepsilon d_s \Phi} \right]^2 \frac{1}{\varepsilon} \mu; \quad A_2 = 1.75 \frac{(1-\varepsilon) \varepsilon_p^2}{\varepsilon^2 d_s \Phi} \rho; \quad q = |\mathbf{v}|.$$

$d_s$  – the equivalent diameter of the particle.

The complexity of calculations according to equations (20) – (22) lies in the fact that in order to calculate the potential  $\varphi$  it is required to know the density  $\rho$ , which depends on the pressure, cross-sectional pore aperture and porosity of the layer. The gas pressure can be determined by the velocity fields  $\mathbf{v}$  and density  $\rho$ . The natural solution to this situation is the use of iterations.

In the case of a plane or axial-symmetric (two-dimensional) flow it is convenient to use the current function  $\psi$ , inserted by the relations:

$$\rho \varepsilon_p u = \partial \psi / \partial y; \quad \rho \varepsilon_p v = -\partial \psi / \partial x \quad (23)$$

and representing the mass consumption of gas.

With the introduction of this function, the continuity equation is satisfied identically, and the condition of the vorticity absence  $\partial u / \partial y - \partial v / \partial x = 0$  leads to the equation for determining  $\psi$

$$\frac{\partial}{\partial x} \left( \frac{1}{\rho \varepsilon_p} \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\rho \varepsilon_p} \frac{\partial \psi}{\partial y} \right) = 0. \quad (24)$$

The boundary conditions of equations (21), (22) and (24) are defined by the profile of the unit and geometry of devices inputting and outputting the gas.

Illustration for using the above described equations of non-vortex flows, comparison of the results of their solution with the data obtained on the basis of the available in the literature of mathematical models of layer gas mechanics, as well as verification of the adequacy of the mathematical model of the potential motion to the real picture of gas flow are described in [2-4, 14-17] and in the works from an extensive bibliography in [3].

It is shown that in terms of uniformity of gas distribution the mathematical model of the potential gas flow in the layer can be effectively used for the selection of optimal geometry parameters of the tuyere device and the blast furnace (furnace profiles). As for the absolute values of velocities and pressures, the exact values of which are necessary for the calculation of heat- and mass- transfers occurring in the layered metallurgical furnaces and units, then the accuracy of their determination characteristic for the potential model should be specifically identified by comparing the solutions of the problems of the vortex and potential movements.



*Vortex flows.* Basically, the calculation of vortex flows is possible on the basis of equations (5) and (12) if we introduce the vector potential of velocity  $\boldsymbol{\Psi}$  by the relation  $\rho \varepsilon_p \mathbf{v} = \text{rot } \boldsymbol{\Psi}$  (details of this approach see in [3] pp. 18–19). Practice of calculations shows, however, that this method is ineffective, because the numerical schemes of finite differences converge very slowly. Therefore, it is appropriate to transform equations (5) and (12), so that they would directly include vorticity of the flow  $\boldsymbol{\omega} = \text{rot } \mathbf{v}$ .

In studying the gases movement through the layer of lump materials we have to deal with two manifestations of the vortex nature of a vortex flow:

- *micro-vorticity*, i.e. with vortices, the scale of which is commensurable with the average sizes of inter-lump space (diameter of a layer particle); the appearance of these vortices is entirely due to gas friction on the surface of particles;
- *macro-vorticity*, i.e. with vortices, the scale of which is commensurable with the furnace working volume; these vortices are conditioned by the geometry of the unit workspace (furnace profile), the presence of combustion zones (circulation zones), etc.

First of all, we shall note that, according to Lagrange's theorem [10] the vortices can not be formed within the area of movement: they are either introduced into it, or formed on its boundaries. The vortices formation on the boundary of flow is due to the sticking phenomenon, i.e., the presence of tangential stresses. On the other hand, setting the condition of sticking as a boundary one for solving the motion equations is possible, if only the latter has the second order, whereas equation (12) is a differential equation of the 1<sup>st</sup> order.

Thus, the introduction of vortices into the area of movement is due to the vortex nature of the gas flow through the tuyeres. As the gas inflows into the developed area of circulation, the vorticity of flow can be enhanced. However, if the gas from the tuyere outflows into the dense layer, the initial vortices are likely to be quickly damped.

However, this situation requires a special consideration, since in this case the interest is arisen, basically, by the principle problem of the possibility of ignoring the vortex nature of the gas flow during the analysis of gas distribution. In this case it is advisable to simplify the equations of gas motion in the layer (5) and (12) to the model of two-dimensional motion of the incompressible gas in the layer of constant porosity. Note, that these simplifications (incompressible gas and the layer of constant porosity) are not fundamental, since using the iterative method for solution of determinative equations, it is not difficult to take into account the changes in these characteristics.

For generalization of the results let us introduce the dimensionless coordinates

$$\tilde{x} = x/R, \quad \tilde{z} = z/R \quad (25)$$

and velocities

$$\tilde{v}_x = v_x/v_0; \quad \tilde{v}_z = v_z/v_0, \quad (26)$$

where  $R$  – furnace radius;  $v_0 = Q_v/R$  – average consumption rate (velocity in the empty shaft);  $Q_v$  – blast consumption. Here the coordinate along the furnace height we denote by  $z$ , to enable solution of three-dimensional problems. Taking into consideration that for the two-dimensional problem of flow vorticity  $\boldsymbol{\omega} = \text{rot } \mathbf{v}$  is a scalar  $\tilde{\omega} = \partial \tilde{v}_x / \partial \tilde{z} - \partial \tilde{v}_z / \partial \tilde{x}$ , from (5) we obtain the first equation of the model:

$$\frac{\partial^2 \tilde{\Psi}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\Psi}}{\partial \tilde{z}^2} = \tilde{\omega}, \quad (27)$$

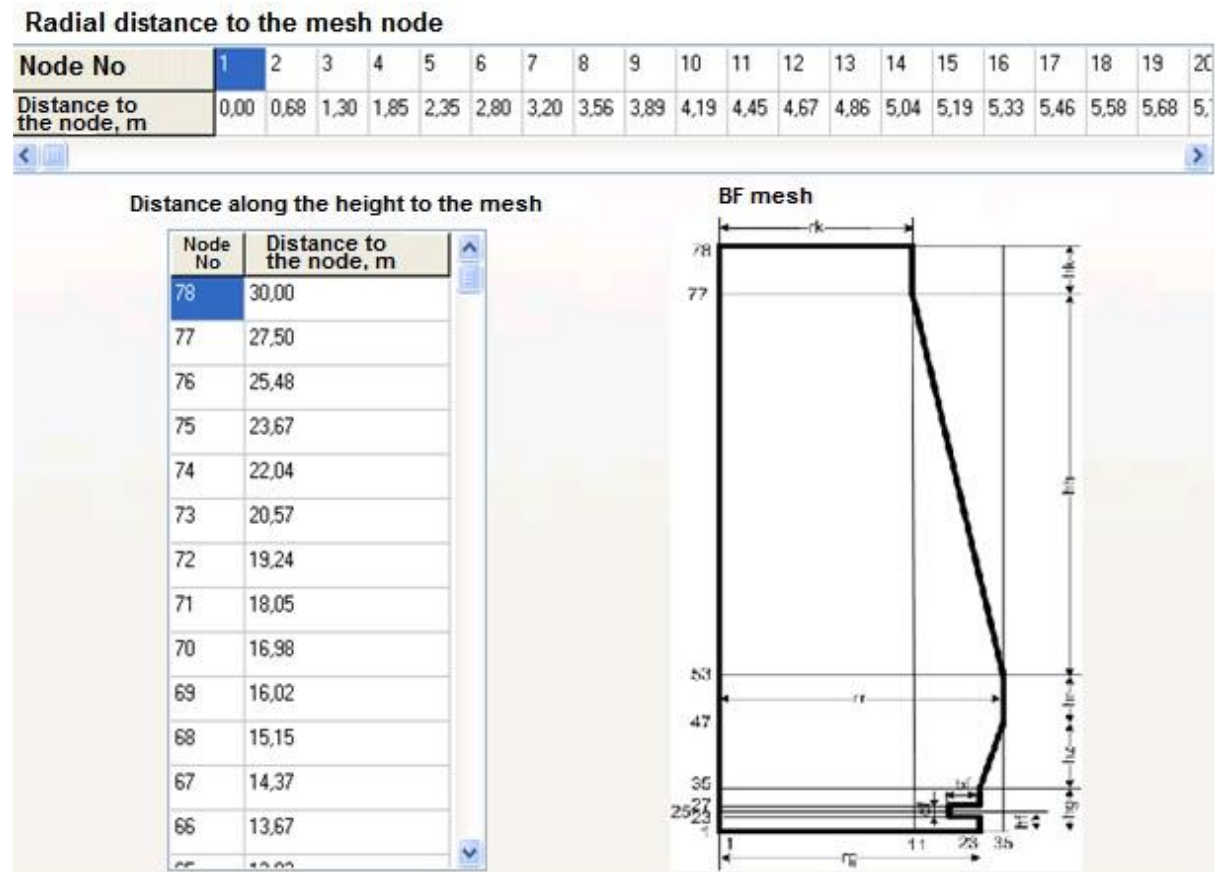
where  $\tilde{\omega} = \omega R / v_0 = \omega R^2 / Q_v$ .

Equation (27) contains two unknown functions, therefore, to solve the problem one more relation is required, namely, the equation of motion (12), which, after simple transformations can be written as Lamb-Gromek equation [10], supplemented by resistance forces

$$\begin{aligned} \frac{\partial}{\partial \tilde{x}} \left( \tilde{\omega} \frac{\partial \tilde{\psi}}{\partial \tilde{z}} \right) - \frac{\partial}{\partial \tilde{z}} \left( \tilde{\omega} \frac{\partial \tilde{\psi}}{\partial \tilde{x}} \right) + M(\tilde{q}) \tilde{\omega} = \\ = -1.75 \left( \frac{\partial \tilde{\psi}}{\partial \tilde{x}} \cdot \frac{\partial \tilde{q}}{\partial \tilde{x}} + \frac{\partial \tilde{\psi}}{\partial \tilde{z}} \cdot \frac{\partial \tilde{q}}{\partial \tilde{z}} \right), \end{aligned} \quad (28)$$

where  $M(q) = A_1 + A_2 q$ .

The system of differential equations (27), (28) describes a vortex motion of the gas in the layered unit of any profile. Therefore, we specify the task and choose the furnace profile shown in Figure 1 (a typical two-dimensional profile of a shaft furnace).



**Figure 1.** Shaft furnace profile and the nodes coordinates of the finite-difference mesh.

For scheme in Figure 1 the boundary conditions for  $\tilde{\psi}$  will be the following:

$$\tilde{\Psi} = 0 \quad \text{if} \quad \begin{cases} \tilde{x} = 0, & 0 \leq \tilde{z} \leq H; \\ 0 \leq \tilde{x} \leq \tilde{x}_{\max}, & \tilde{z} = 0; \\ \tilde{x} = \tilde{x}_{\max}, & 0 \leq \tilde{z} \leq \tilde{L} - \tilde{r}_t; \end{cases} \quad (29)$$

$$\tilde{\Psi} = 1 \quad \text{if} \quad \tilde{x} = \tilde{x}_{\max}, \quad \tilde{L} + \tilde{r}_t \leq \tilde{z} \leq H; \quad (30)$$

Here the index  $t$  means the lance blast furnace.

$$\frac{\partial \tilde{\Psi}}{\partial \tilde{z}} = 0 \quad \text{if} \quad 0 \leq \tilde{x} \leq \tilde{x}_{\max}, \quad \tilde{z} = H. \quad (31)$$

The value  $\tilde{\Psi}$  on the cross-section of the tuyere, i.e. if  $\tilde{x} = \tilde{x}_{\max}$ ,  $\tilde{L} - \tilde{r}_t \leq \tilde{z} \leq L + \tilde{r}_t$  is determined by the mode of blast motion in the tuyere. For laminar mode the velocity profile is described by a quadratic parabola, and the ratio of the average velocity to the maximal equals to 0.5. For the conditions Figure 1

$$\tilde{v}_x = -\frac{3}{4} \tilde{r}_t^3 [\tilde{L}^2 - \tilde{r}_t^2 - \tilde{z}(2\tilde{L} - \tilde{z})].$$

Since  $\tilde{v}_x = \partial \tilde{\Psi} / \partial \tilde{z}$ , then taking into account the fact that the below the tuyere  $\tilde{\Psi} = 0$ , after integration at  $\tilde{x} = \tilde{x}_{\max}$ ,  $\tilde{L} - \tilde{r}_t \leq \tilde{z} \leq L + \tilde{r}_t$  we find

$$\tilde{\Psi} = 0.75 \tilde{r}_t^3 \left[ (\tilde{L} + 2\tilde{r}_t)(\tilde{L} - \tilde{r}_t)^2 - (\tilde{L}^2 - \tilde{r}_t^2)\tilde{z} + \tilde{L}\tilde{z}^2 - \frac{1}{3}\tilde{z}^3 \right],$$

i.e. the  $\tilde{\Psi}$  profile on the tuyere is a cubic parabola.

In case of turbulent gas flow in the tube the following equation best agrees with the experimental data throughout the turbulent core of the flow up to the pipe axis [21]:

$$\tilde{v}_x = 0.2 \frac{\tilde{d}_t}{\text{Re}^{1/8}} \left\{ 5.5 + 2.5 \ln \left[ 0.2 \text{Re}^{7/8} \frac{\tilde{z} - \tilde{L} + \tilde{d}_t/2}{\tilde{d}_t} \times \frac{1.5[1 + (\tilde{L} - \tilde{z})/\tilde{r}_t]}{1 + 2[(\tilde{L} - \tilde{z})/\tilde{r}_t]} \right] \right\}. \quad (32)$$

Here  $\tilde{d}_t = 2\tilde{r}_t$ . Then if  $\tilde{x} = \tilde{x}_{\max}$ ,  $\tilde{L} - \tilde{r}_t \leq \tilde{z} \leq L + \tilde{r}_t$  we get

$$\tilde{\Psi} = \int_{\tilde{L} - \tilde{r}_t}^{\tilde{L} + \tilde{r}_t} \tilde{v}_x(\tilde{x}_{\max}, \tilde{z}) d\tilde{z}.$$

Within the laminar sublayer the relations are valid:

- if  $\tilde{z} < \tilde{L} - \tilde{r}_t + [\tilde{d}_t / (0.04 \text{Re}^{7/8})]$   $\tilde{v}_x = 0.04 \text{Re}^{3/4} (\tilde{z} - \tilde{L} + \tilde{r}_t) / \tilde{d}_t^2$ ;

- if  $\tilde{z} > \tilde{L} - \tilde{r}_t - [\tilde{d}_t / (0.04 \text{Re}^{7/8})]$   $\tilde{v}_x = 0.04 \text{Re}^{3/4} (\tilde{L} + \tilde{r}_t - \tilde{z}) / \tilde{d}_t^2$ .

Let us move to the boundary conditions on vorticity  $\tilde{\omega}$ . If  $\tilde{x} = \tilde{x}_{\max}$  and  $\tilde{L} - \tilde{r}_t \leq \tilde{z} \leq \tilde{L} + \tilde{r}_t$ , i.e. at the tuyere cross-section  $\tilde{v}_z = \partial \tilde{v}_z / \partial \tilde{x} = 0$  and  $\tilde{\omega} = \partial \tilde{v}_x / \partial \tilde{z}$ . For laminar mode  $\tilde{\omega} = 1.25(\tilde{L} - \tilde{z}) / r_t^3$ .

In case of turbulence mode at the lower tuyere stage  $\tilde{\omega} = 0.04 \text{Re}^{3/4} / \tilde{d}_t^2$ , and at the upper  $\tilde{\omega} = -0.04 \text{Re}^{3/4} / \tilde{d}_t^2$ . For the flow core the vorticity is determined by the equation differentiation (32)  $\tilde{z}$ :

$$\tilde{\omega} = \partial \tilde{v}_x / \partial \tilde{z} = 2.24(\tilde{L} - \tilde{z}) \tilde{r}_t / \{ \text{Re}^{1/8} [\tilde{r}_t^2 - (\tilde{L} - \tilde{z})^2] [\tilde{r}_t^2 + 2(\tilde{L} - \tilde{z})^2] \}.$$

On the furnace axis if  $\tilde{x} = 0$  and  $0 \leq \tilde{z} \leq \tilde{H}$   $\tilde{v}_x = \partial \tilde{v}_x / \partial \tilde{z} = 0$ , i.e.  $\tilde{\omega} = -\partial \tilde{v}_z / \partial \tilde{x}$ . However, we consider a symmetric problem, and in this case the profile of vertical component of velocity is symmetrical. But then  $\partial \tilde{v}_z / \partial \tilde{x} = 0$  and  $\tilde{\omega}(0, \tilde{z}) = 0$ .

Expression (28) with regard to the continuity equation  $\partial \tilde{v}_x / \partial \tilde{x} + \partial \tilde{v}_z / \partial \tilde{z} = 0$  can be written as:

$$\tilde{v}_x \frac{\partial \tilde{\omega}}{\partial \tilde{x}} + \tilde{v}_z \frac{\partial \tilde{\omega}}{\partial \tilde{z}} + (A_1 + A_2 \tilde{q}) \tilde{\omega} = A_2 \left( \tilde{v}_z \frac{\partial \tilde{q}}{\partial \tilde{x}} - \tilde{v}_x \frac{\partial \tilde{q}}{\partial \tilde{z}} \right).$$

On the furnace wall if  $\tilde{x} = \tilde{x}_{\max}$  and  $0 \leq \tilde{z} \leq \tilde{L} - \tilde{r}_t$  or  $\tilde{L} + \tilde{r}_t \leq \tilde{z} \leq \tilde{H}$   $\tilde{v}_x = 0$  and  $\tilde{q} = |\tilde{v}_z|$ . Thus,

$$\tilde{v}_z \frac{\partial \tilde{\omega}}{\partial \tilde{z}} + (A_1 + A_2 |\tilde{v}_z|) \tilde{\omega} = A_2 \tilde{v}_z \frac{\partial |\tilde{v}_z|}{\partial \tilde{x}}. \quad (33)$$

With values  $\tilde{\omega}$  on the tuyere cross-section, solving equation (33), we can determine the vorticity along the furnace wall upward from the tuyere to the charge level and downward from the tuyere to the bottom. At the bottom if  $\tilde{z} = 0$  and  $0 \leq \tilde{x} \leq \tilde{x}_{\max}$   $\tilde{v}_z = 0$  and  $\tilde{q} = |\tilde{v}_x|$ . Then

$$\tilde{v}_x \frac{\partial \tilde{\omega}}{\partial \tilde{x}} + (A_1 + A_2 |\tilde{v}_x|) \tilde{\omega} = -A_2 \tilde{v}_x \frac{\partial |\tilde{v}_x|}{\partial \tilde{z}}. \quad (34)$$

At the charge level the relation (34) is also valid.

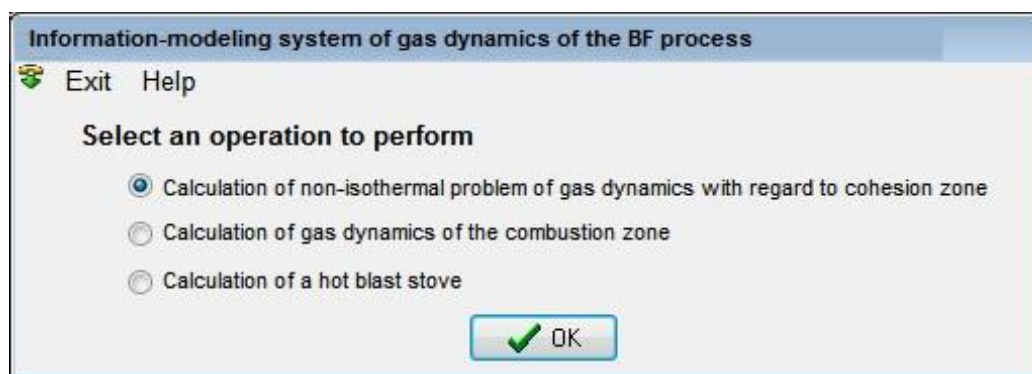
The finite-difference approximation of the above stated equations in the variables “vorticity - flow function” is a numerical model of the gas vortex flow in the layer of the lump material. Computing practice has shown that it is more rational to solve the equations of this model by the method of successive upper relaxation (SUR) [2-4, 11, 18-24].

#### 4. The example of solution to the equations of a model of vortex problem for the blast furnace

For the operational conditions of blast furnace with height (along the layer) 30 m and a belly diameter 6.55 m the information-modeling system of gas dynamics of the BF process was developed. The

system included a solution to the problem of non-isothermal vortex flow in the blast furnace at a given porosity field taking into account the cohesion area, the problem of gas dynamics of the combustion zone and a program for calculating the hot blast stove. The software implementation of the system was carried out in Delphi 7 and Compaq Visual Fortran. Unfortunately, the second software package does not function in Windows 7; therefore, it is impossible to illustrate the operation of the system in it. However, the application contains all program modules of the system in Compaq Visual Fortran, since, for example, in Windows XP this package functions perfectly.

In Delphi the system is started by command **Gazodin\_DP.exe**, which opens the initial form (Figure 2). When choosing a problem to be solved, for example, "Calculation of non-isothermal problem of gas dynamics with regard to cohesion zone", the form with geometrical and regime parameters of the analyzed BF is opened (Figure 3).



**Figure 2.** The form for system start-up.

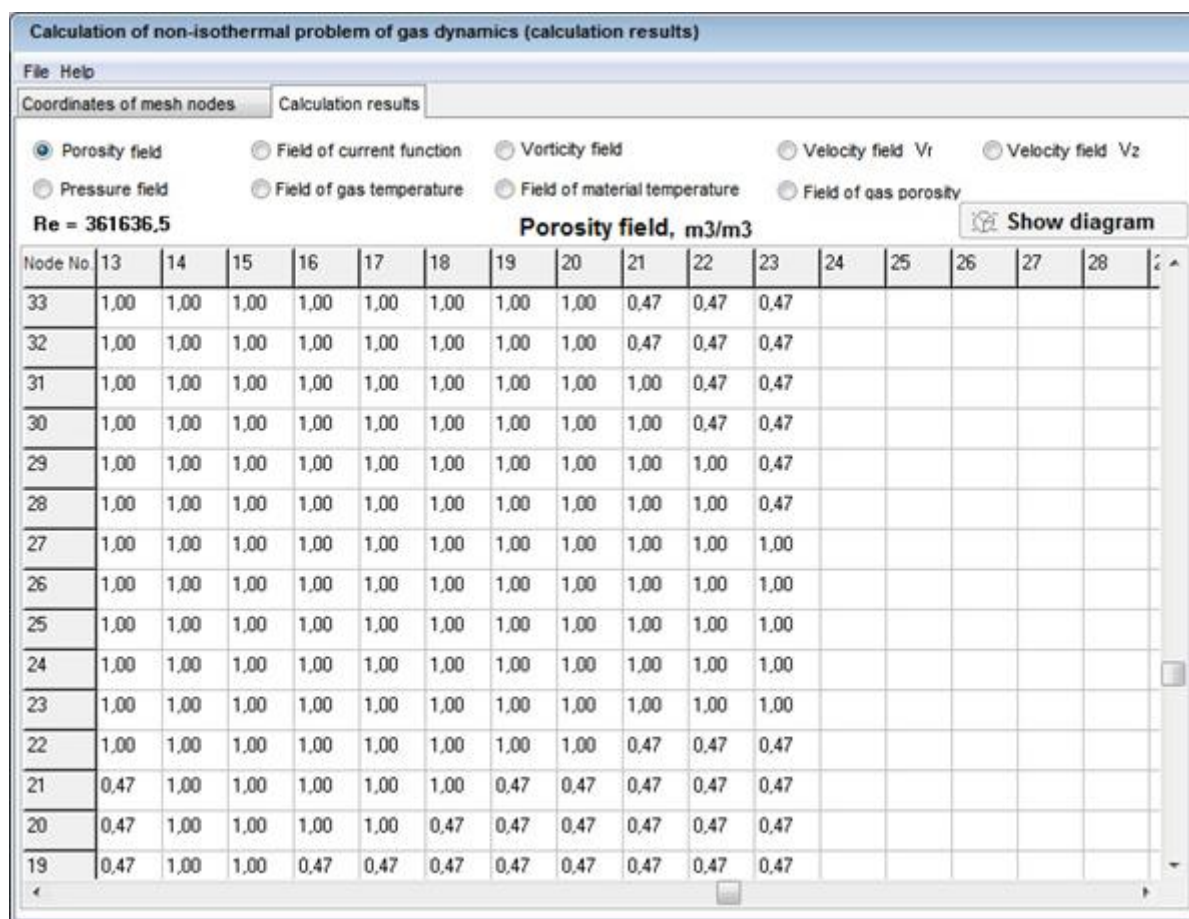
rk, m	4.45	rr, m	6.55	rg, m	6	hk, m	2.5
hh, m	19.6	hr, m	2.3	hz, m	3.4	hg, m	2.2
hf, m	1.5	df, m	0.18	bf, m	0	nf, pieces	28
q0, m/s	90	dz1	1	dz2	1	dz3	1
dz4	1	dz5	1	l, m	1.25	akf, fraction	1.2
ro0, kg/m3	1.293	mu0, 10 <sup>-6</sup> *H*c/m2	17.08	dk, m	0.03	form, fraction	0.9
dpsi	0.0025	dom	0.005	dpg	0.001	dtg	0.001
p0, 10 <sup>5</sup> *Pa	4	tg0, C	1400	tm0, C	100	lmd0, W/(m*K)	0.0244
cv0, J/(m3*K)	1002	rom0, kg/m3	1500	qm, kr/c	55		

**Figure 3.** Setting BF operational parameters.

In this form  $r_k$ ,  $r_r$ ,  $r_g$  – are radii of the furnace mouth, belly and hearth;  $h_k$ ,  $h_h$ ,  $h_r$ ,  $h_z$ ,  $h_g$ ,  $h_f$  – are heights of the furnace mouth, shaft, belly, boshes and the tuyeres arrangement horizon. Similarly  $d_k$  – diameter of charge lumps;  $d_f$  – the tuyres diameter;  $q_0$  – velocity of the blast outflow from the tuyere;  $d_{psi}$ ,  $d_{om}$ ,  $d_{pg}$ ,  $d_{tg}$  – precision of the calculations (iterations convergence criteria) of the flow function, vorticity, gas pressure and gas temperature. The remaining values are standard and do not require explanation.

As it can be seen from the data in Figure 3, the form offers *two* options. Clicking “File” you can enter the same data for another furnace or enter other values for the same furnace. By clicking “Display Results” the calculation according to the equations of numerical model is performed and the results are shown in the tabular form (Figure 1). The first form contains the nodes coordinates of finite difference mesh, which shows that along the furnace height (up to 30 m) 78 nodes are distinguished and along the furnace radius in the belly zone (up to 6.55 m) – 35 nodes. Note that the mesh cell dimensions are not the same – they are minimal near the combustion zone increase on approaching to the charge level. This form is shown in Figure 1 and is not given here.

The form with the calculation results reveals the distribution of nine gas-dynamic parameters in the layer volume (Figure 4).

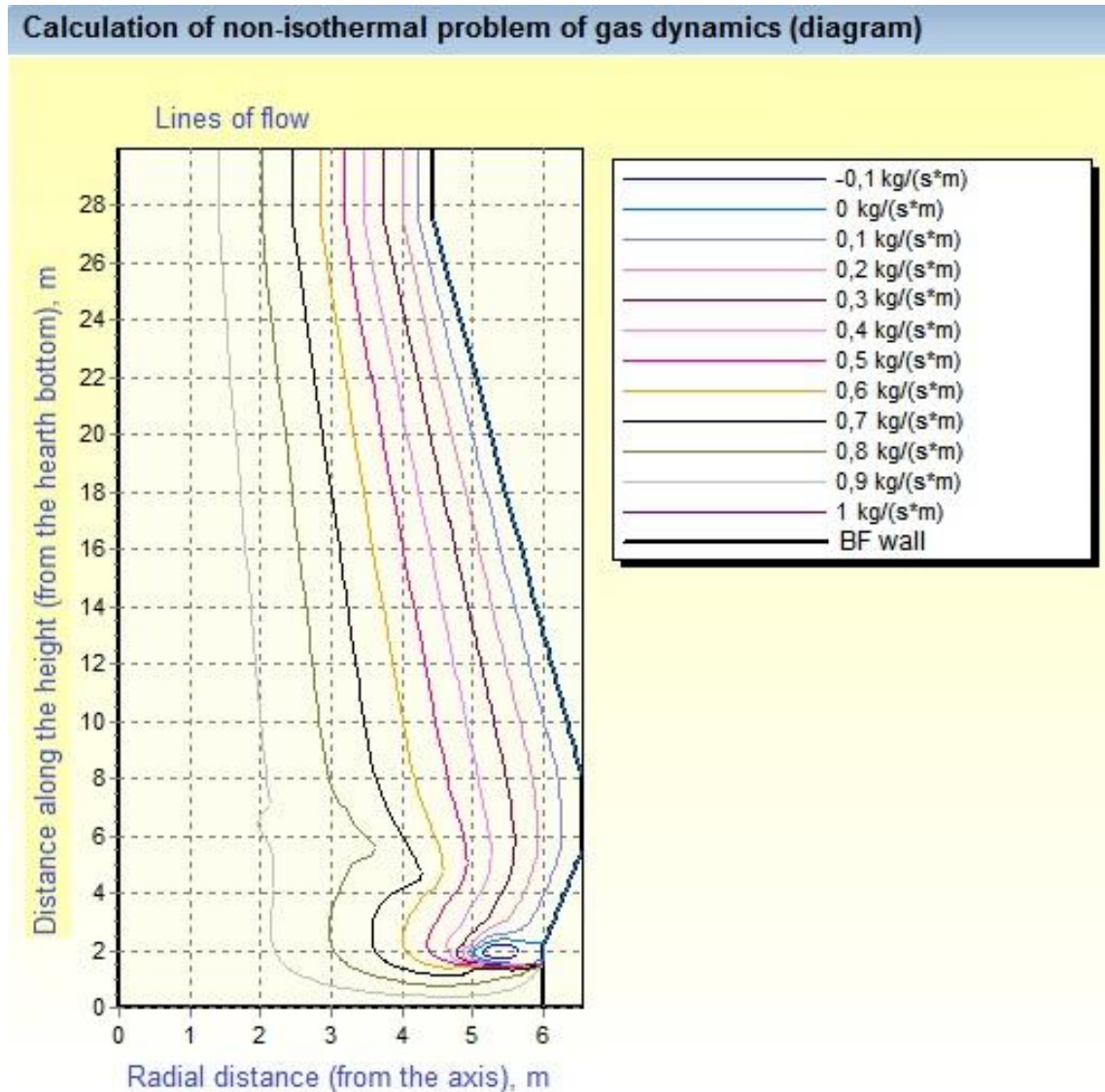


**Figure 4.** The form contains the results of the solution to the non-isothermal problem of gas dynamics.

Three parameters (flow function, gas temperature and material temperature) can be displayed graphically after clicking “Show diagram”. After clicking in the start form “Calculation of gas dynamics of the combustion zone” the form with combustion zone parameters is opened (Figure 6). Note that the theory of operation of the BF combustion zone and the detailed output of determining equations of gas dynamics and heat-transfer are presented in our works [25, 26]. As for the notations

in Figure 6, then, in our opinion, should be explained only parameter 1 tuyere height, i.e. the distance of the tuyere cross-section to the inner wall of hearth,  $t_{rec}$  – the recycle temperature or, in other words, the gas leaving the tuyere, but returning to the combustion zone due to the flow vorticity, and PartRec – is the recycle portion from the total amount of the tuyere gas.

Clicking the button “Display results” the form with the tables of all combustion zone characteristics is opened. Perhaps the option of “File” is of particular interest, where the graph of changes in the combustion zone parameters can be obtained (Figure 8 and 9).



**Figure 5.** Graph of the gases flow in the blast furnace.



**Calculation of gas dynamics of the combustion zone (input of initial data)**

File Help

$l$ , m	<input type="text" value="1.3"/>	$V_{blast}$ , m <sup>3</sup> /s	<input type="text" value="35"/>	$t_{blast}$ , C	<input type="text" value="1200"/>	$W_{coal}$ , g/m <sup>3</sup>	<input type="text" value="0"/>
$W_{O_2}$ , m <sup>3</sup> /m <sup>3</sup>	<input type="text" value="0.05"/>	$W_{st}$ , g/m <sup>3</sup>	<input type="text" value="24"/>	$W_{ng}$ , m <sup>3</sup> /m <sup>3</sup>	<input type="text" value="0"/>	$p$ , atm	<input type="text" value="3.5"/>
$R_{oob}$ , kg/m <sup>3</sup>	<input type="text" value="1050"/>	$\beta$ , fraction	<input type="text" value="0.85"/>	$d_g$ , m	<input type="text" value="9.5"/>	$d_f$ , m	<input type="text" value="0.16"/>
$n_f$ , pieces	<input type="text" value="24"/>	$K_{si}$ , fraction	<input type="text" value="0.15"/>	$t_{cocks}$ , C	<input type="text" value="1600"/>	$\epsilon$ , fraction	<input type="text" value="0.5"/>
$Por$ , fraction	<input type="text" value="0.55"/>	$O_{2crit}$ , fraction	<input type="text" value="0.03"/>	$dk_{ob}$ , m	<input type="text" value="0.045"/>	$f_{form}$ , fraction	<input type="text" value="0.875"/>
$t_{rec}$ , C	<input type="text" value="1977"/>	$PartRec$ , fraction	<input type="text" value="0.2"/>	$X_{in}$ , m	<input type="text" value="1.6"/>	$X_{out}$ , m	<input type="text" value="1.8"/>
$d_{cd}$ , mm	<input type="text" value="0.1"/>	$\rho_{cd}$ , kg/m <sup>3</sup>	<input type="text" value="1320"/>	$por_{cd}$ , fraction	<input type="text" value="0.12"/>	$F_{ud}$ , m <sup>2</sup> /kg	<input type="text" value="3000"/>
$C_{r_{cd}}$ , wt%	<input type="text" value="74.5"/>	$H_{r_{cd}}$ , wt%	<input type="text" value="4.82"/>	$O_{r_{cd}}$ , wt%	<input type="text" value="8.01"/>	$Vol_{a_{cd}}$ , wt%	<input type="text" value="38.7"/>
$Ar_{cd}$ , wt%	<input type="text" value="7.1"/>	$V_x$ , m/s	<input type="text" value="75"/>	$V_y$ , m/s	<input type="text" value="0"/>	$t_{0_{cd}}$ , C	<input type="text" value="1050"/>

Comments:

Figure 6. Operational parameters of BF combustion zone.

**Calculation of gas dynamics of the combustion zone (calculation results)**

File Help

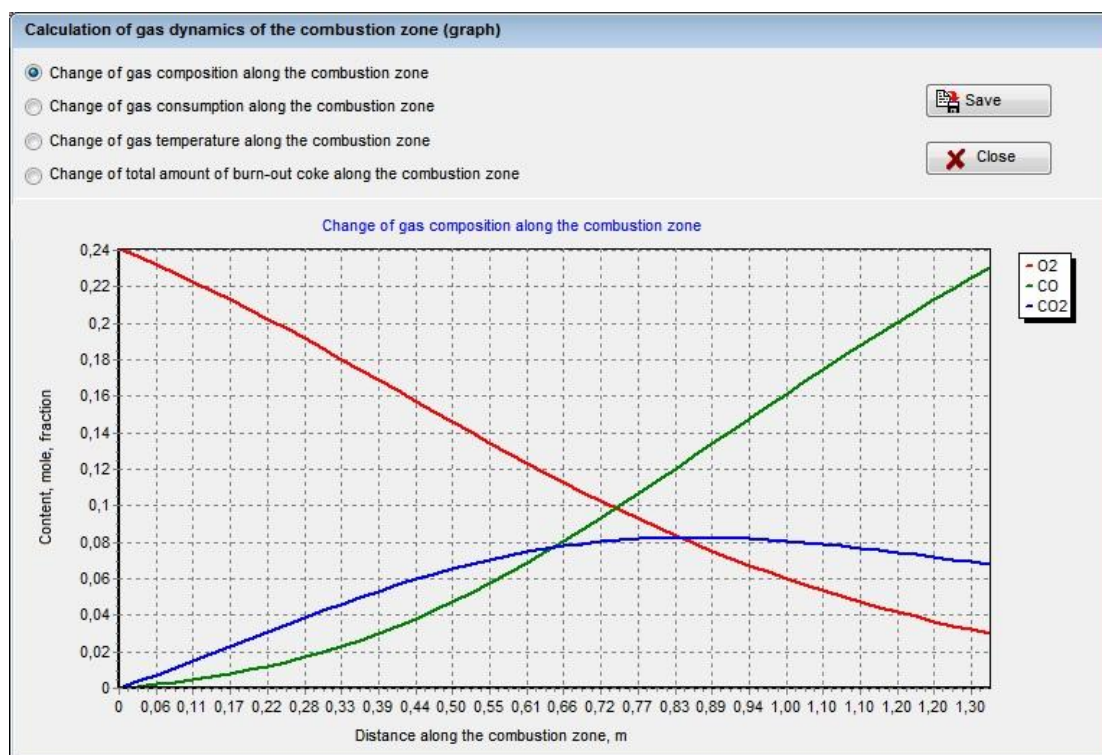
☒ Gas temperature, C
 ☒ CO<sub>2</sub> content, mole, fraction
 ☒ Total amount of the burnout coke, kg
 ☒ H<sub>2</sub>O content, mole, fraction
 ☒ Tuyere gas consumption, mole/(m<sup>3</sup>\*s)
 ☒ CO content, mole, fraction
 ☒ O<sub>2</sub> content, mole, fraction
 ☒ H<sub>2</sub> content, mole, fraction

Average temperature of the combustion zone C = 1990,29

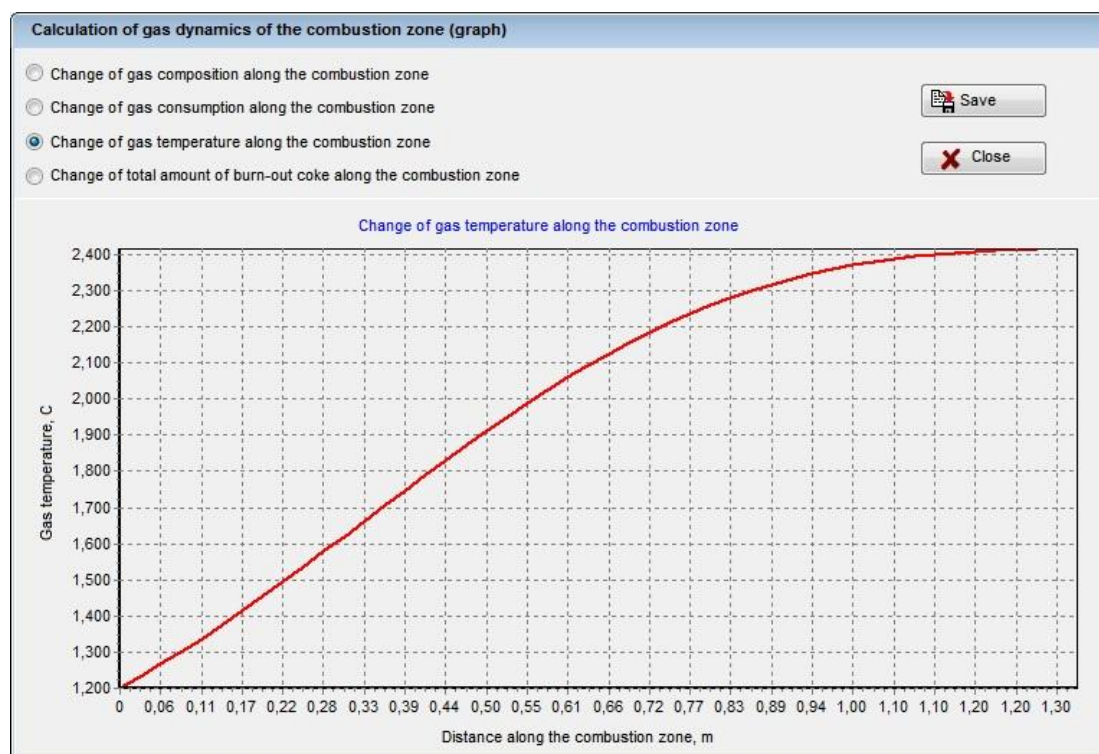
Node No.	1	2	3	4	5	6	7
Gas temperature, C	1200,00	1232,19	1265,81	1300,82	1337,16	1374,75	1413,49
Total amount of the burnout coke, kg	0,00	430,95	445,61	460,88	476,67	492,92	509,50
Tuyere gas consumption, mole/(m <sup>3</sup> *s)	3496,63	3477,87	3460,35	3443,86	3428,36	3413,87	3400,48
O <sub>2</sub> content, mole, fraction	0,24077	0,23646	0,23200	0,22741	0,22267	0,21780	0,21280
CO <sub>2</sub> content, mole, fraction	0,00000	0,00369	0,00745	0,01127	0,01514	0,01904	0,02297
H <sub>2</sub> O content, mole, fraction	0,02765	0,02763	0,02762	0,02760	0,02758	0,02756	0,02754
CO content, mole, fraction	0,00000	0,00101	0,00213	0,00338	0,00477	0,00633	0,00806
H <sub>2</sub> content, mole, fraction	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Figure 7. Characteristics of BF combustion zone.





**Figure 8.** Change in the tuyere gas composition along the combustion zone.



**Figure 9.** Changes in the tuyere gas temperature along the length of the blast chamber.

Calculations show that the large-scale vortices are intensively damped by the non-deformable layer, and the decrease intensity of vorticity is determined mainly by the gas Reynolds number. With an increase of number Re the area dimensions, where it is necessary to take into account the vortex nature of the gas flow in the layer, increases. If to accept  $\tilde{\omega} = 0.05$  as a boundary of vortex area, then the layer volume, occupied by the vortex flows, is 41%. It is clear that in these cases the vortex nature of the gas flow in the layer cannot be ignored. Note that the zone of two-dimensional gas flow takes in this case 34% of the layer volume, i.e. flow vorticity covers the lower part of one-dimensional flow (~12.3% of its volume).

## 5. Conclusions

Thus, the mathematical model can be used for any metallurgical unit. Of course, in each case, the equations system of the model, and the algorithm of its solution will have its own characteristics, but the methodology of constructing the model remains unchanged.

## 6. Acknowledgements

The work was supported by Act 211 Government of the Russian Federation, contract № 02.A03.21.0006.

## 7. References

- [1] Matros Yu Sh 1982 *Non-stationary Processes in Catalytic Reactors* (Novosibirsk: Nauka) p 228
- [2] Kitaev B I, Yaroshenko Yu G, Sukhanov E L, Ovchinnikov Yu N, and Shvydkiy V S 1978 *Heating Engineering of BF Process* (M.: Metallurgiya) p 248
- [3] Gordon Ya M, Bokovikov B A, Shvydkiy V S and Yaroshenko Yu G 1989 *Thermal Operation of Shaft Furnaces and Units With a Dense Layer* (M.: Metallurgiya) p120
- [4] Gordon Ya M, Maksimov E V and Shvydkiy V S 1989 *Mechanics of Materials and Gases Motion in the Shaft Furnaces* (Alma-Ata: Nauka) p 144
- [5] Lykov A V 1972 *Heat- and Mass-transfer (Reference Book)* (M.: Energiya) chap 5 pp 338–467
- [6] Slattery J S 1978 *The theory of Momentum Transfer, Energy and Mass in Continuous Media* (M.: Energiya) p 448
- [7] Smirnov V I 1955 *Course of Higher Mathematics* (M.: Nauka) vol 1 p 480
- [8] Myshkis A D 1969 *Lectures on Higher Mathematics* (M.: Nauka) p 640
- [9] Shvydkiy V S, Ladygichev M G, Shavrin V S 2001 *Mathematical Methods of Thermal Physics: Textbook for Universities* (M.: Mashinostroyeniye) p 232
- [10] Serrin J 1963 *Mathematical Foundations of Classical Fluid Mechanics* (M.: IL) p 256
- [11] Shvydkiy Yu G, Yaroshenko Ya M, Gordon V S, Shavrin V S and Noskov A S 2003 *Fluid Mechanics: Learners' Guide for Universities* (M.: IKTs "Akademkniga") ed V S Shvydkiy p 464
- [12] Shlikhtin G *Tethory of the Boundary Layer* 1974 (M.: Nauka) p 586
- [13] Prager S 1961 *Phys. of Fluids* **4** 1477–85
- [14] Goldshtik M A 1984 *Transfer Processes in the Grain Layer* (Novosibirsk: Inst. of Thermal Physics SB RAS) p 164
- [15] Bogdandi L F and Engel G D 1971 *Iron Ore Reduction* (M.: Metallurgiya) p 520
- [16] Brauer H 1960 *Chem. Ing. Tech.* **32** 585 – 590
- [17] Eshar R, Bredegeft R, Mavrodis M et al 1971 *Ferrous Metals* **12** 121–40
- [18] Patankar S 1984 *Numerical Methods for Solving Problems of Heat Transfer and Fluid Dynamics* (M.: Energoatomizdat) p 152

- [19] Shi D 1991 *Numerical Methods in Heat Transfer Problems* (M.: Mir) p 544
- [20] Frank-Kamenitsky DA 1987 *Diffusion and Heat Transfer in Chemical Kinetics* (M.: Nauka) p 492
- [21] Keyns V M 1972 *Convective Heat and Mass Transfer* (M.: Energiya) p 448
- [22] Kheygman L and Yang D 1986 *Applied Iterative Methods* (M.: Mir) p 448
- [23] Samarsky A A and Nikolaev E S 1978 *Methods for Solving Grid Equations* (M.: Nauka) p 512
- [24] Gosmen A D, Pan V M, Ranchel A K, Spolding D B and Volshteyn M 1972 *Numerical Methods for the Study of Viscous Fluid* (M.: Mir) p 328
- [25] Spirin N A, Ovchinnikov Yu N, Shvydkiy V S and Yaroshenko Yu G 1995 *Heat transfer and efficiency of blast furnace* (Ekaterinburg: UrSTU) ed Yu G Yaroshenko p 243
- [26] Spirin N A, Shvydkiy V S, Lobanov V I and Lavrov V V 1999 *Introduction to the systematic analysis of thermal process industry: Learner' Guide for Universities* (Ekaterinburg: UrSTU – UrPU) p 205